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The Application of Fourier Analysis in Different Fields

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Abstract: Fourier Analysis is an important mathematical tool proposed by Fourier in the early nineteenth century. It is commonly used in our life even though someone may not know it and has been applied into many fields and played different roles, ranging from processing signals, analysing images to solving partial differential equations. This work would summarize the applications and roles of Fourier Analysis and show the future forecast of it.

Keywords: Fourier Analysis, application, expectation

1. Introduction

As a common mathematical tool, graphs can visually demonstrate the connections between variables. Analysing graphs is significant, since it would help us to find the regulation of the variables. However, finding the rule of the variables just by observing the graphs with the naked eyes may not be accurate and efficient enough, so we need some other mathematical tools to assist us.

Fourier Analysis is one of the core tools for image processing, which could reveal the frequency characteristics and energy distribution of the signal. As early as 1807, as Fourier want to solve the partial differential functions derived in thermal conduction, he first proposed that any function could be expressed as a trigonometric series [1]. This idea was later refined by other scholars. Dirichlet proved the convergence of Fourier series under specific conditions which is a sufficient and unnecessary conditions of Fourier Analysis [2]. These conditions were further refined by Jordan, Lebeau, and others, and were finally named after Dirichlet. Based on his work, Riemann conducted deeper studies of Fourier Analysis and expanded the applicability of it, so that more types of functions can be expanded into Fourier series [3], which made profound effect on the development of Fourier Analysis. Later, Lebesgue redefined integration, overcame the limitation of Riemann integral and made Fourier Analysis have wider field of application [4]. Then Fourier Analysis was further expanded through the development of functional analysis and generalized functions, being applied to various fields. For instance, it can analyse the characteristics of signals and capture weak signals, making people's work more efficient [5]. It can also help to distinguish marine-continental transitional shale, which is more suitable for complex marine-continental transitional shale assise [6]. Moreover, it can analyse and detect food in a quick and non-destructive way and has its advantages especially in the aspect of controlling food quality [7]. Fourier analysis enables the

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conversion between the time domain and the frequency domain, significantly enhancing the efficiency and accuracy of the analysis.

In this paper, we will first introduce correlation formulas of Fourier Analysis in the second part. Then, in the third part, we will explore and summarize the application of Fourier Analysis in multiple fields, ranging from distinguishing variety, processing signals to testing products, which would be conductive to those who are interested in Fourier Analysis to understand the role of Fourier Analysis. Additionally, in the last part, we will make expectation about the applicability of Fourier Analysis, hoping that it would be applied into more fields and make people's life much more convenient, more efficient and more intelligent.

2. Fourier Analysis

Fourier Analysis includes Fourier Series and Fourier Transform.

Fourier Series (FS) can be used to express periodic functions as infinite sine and cosine functions [8]. Here are the formulas.

The first one is the real coefficient form, which is also called trigonometric series.

$f(t)$ is a continuous function

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega t)), \quad \omega = \frac{2\pi}{T}$$

where $a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)dt, \quad b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt$

It also has complex exponential form, which is more generally used.

$$f(t) = \sum_{n=1}^{\infty} c_n e^{jn\omega t}, \quad c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega t}dt, \quad \omega = \frac{2\pi}{T}$$

where j is the unit imaginary number.

Fourier Transform (FT) can be divided into Continuous Fourier Transform (CFT) and Discrete Fourier Transform (DFT). It is used to express aperiodic functions, describing the amplitudes of different frequencies of sine or cosine functions contained in the function. Here are the formulas.

Continuous Fourier Transform (CFT):

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

where j is the unit imaginary number.

Inverse Fourier Transform (IFT):

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

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where j is the unit imaginary number.

Fourier Transformation Infrared Spectroscopy (FTIR) is a technique that determines the infrared spectrum by measuring the interference pattern and performing a Fourier transformation on it. It has the characteristics of high detection sensitivity, high measurement accuracy, high resolution, fast measurement speed, low astigmatism and wide spectral band.

Discrete Fourier Transform (DFT):

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}, \quad k = 0,1,\ldots,N-1$$

Inverse Discrete Fourier Transform (IDFT):

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}, \quad n = 0,1,\ldots,N-1$$

Fast Fourier Transform (FFT):

Fast Fourier Transform (FFT) is an efficient algorithm to compute DFT. Its core idea relies on a divide-and-conquer approach and exploiting the symmetry of twiddle factors to reduce computational complexity from $O(N^2)$ to $O(logN)$. While FFT does not alter the mathematical definition of the DFT, it dramatically accelerates computation through optimized steps, making it indispensable in signal processing, image analysis, and other fields. The key steps include bit-reversal reputation, butterfly operations and progressive result merging.

Sparse Fourier Transform (SFT):

Sparse Fourier Transform is an efficient algorithm especially designed for frequency domain sparse signals, sparse signals, where most of the frequency components are zero or nearly zero. It is a kind of reduced form of Discrete Fourier Transform, and is 10 to 100 times faster than the traditional Fast Fourier Transform. Its core concept is to calculate only the significant frequency components through frequency-domain hashing and down sampling, thereby reducing the complexity from $O(logN)$ to $O(klogN)$, where k is the number of non-zero components. The key steps include frequency hashing, local spectrum computation and dominant frequency recovery.

Two-Dimensional Discrete Fourier Transform (2D-DFT):

Given a discrete two-dimensional signal (matrix) $x[m,n]$ with dimensions M×N, its 2D-DFT is defined as:

$$X[k,l] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x[m,n] \bullet e^{-i2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}, \quad k = 0,1,\ldots.M-1,\ l = 0,1,\ldots,N-1$$

where i is the unit imaginary number.

Two-Dimensional Inverse Discrete Fourier Transform (2D-IDFT):

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$$x\left[m,n\right]=\frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}X\left[k,l\right]\bullet e^{i2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

Short-Time Fourier Transform (STFT):

Short-Time Fourier Transform is used to analyse the time-frequency characteristics of the signals that are not stable, that is, signals whose frequency would change as time over time.

$$X\left(t,f\right)=\int_{-\infty}^{\infty}x\left(\tau\right)\omega\left(\tau-t\right)e^{-j2\pi f\tau}d\tau$$

where$x(\tau)$ is the input signal, $\omega(\tau-t)$is a window function, $j$is the unit imaginary number.

3. Application of Fourier Analysis

3.1 Application in distinguishing variety

Fourier Transformation is one of the core tools in processing signals. It can transform time domain signals into frequency domain signals [9]. In the nature, there are many objects seems to be very similar so that sometimes people cannot distinguish these objects quickly and accurately with naked eyes. Fourier Transformation and its derivative technology could help people to distinguish variety.

Fourier Transformation Infrared Spectroscopy (FTIR) is one of the most significant technologies in spectrum analysis. Infrared spectroscopy technology is a rapid, non-destructive and environmentally friendly detection technique that has emerged in recent years [10]. It utilizes the absorption characteristics of infrared light in specific spectral intervals to comprehensively reflect the compositional differences among samples [11]. With the advantage of simple operation, fast analysis speed, good test reproducibility and non-destructive testing, infrared spectroscopy has been applied to the analysis and detection in many fields. Wang et al. (2024) has found that, by utilizing the distinctive near-infrared spectroscopy and mid-infrared spectroscopy spectral absorption characteristics of different fish species with specific wavelength bands, combined with other qualitative analytical model, it is able to distinguish three Sciaenidae species rapidly and accurately.

Two-Dimensional Discrete Fourier Transform (2D-DFT) is a core tool to deal with two-dimensional discrete signals, like images and matrix. It could analyse and process multidimensional signals by transforming spatial domain signals into frequency domain signals. Li et al. [6] has done a study. Their work found that the traditional method used to identify shale lamina was limited, so this work proposed a new identification method based on Two-Dimensional Discrete Fourier transform and principal component analysis. This work used 2D-DFT to transform the images of shale lamina into the frequency domain image and extracted the characteristics of the frequency domain image by taking the principal component analysis technology. Then this work applied the method into the researched area and found that this method was more suitable for complex and diverse marine-terrestrial transitional shale formations compared to conventional methods. The method achieves over 90% identification accuracy but may fail in highly heterogeneous rocks or those with abundant dispersed fine particles.

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To sum up, Fourier Transformation could transform aperiodic signals into frequency domain signals, which would be more convenient to extract and compare the difference between objects. It helps people to distinguish variety in a more efficient and accurate way.

3.2 Application in processing signals

Signals can be seen everywhere in our daily life. Processing signals accurately and rapidly is very important and necessary. The core of Fourier Analysis is to transform signals from time domain into frequency domain, helping people to analyse the characteristics of complex signals.

Sparse Fourier Transform (SFT) is a kind of efficient Fourier Analysis algorithms specifically designed for sparse signals that only have a few components in the frequency domain. Cao [5] proposed a system design based on Sparse Fourier Transform (SFT) to address the problem of weak signal capture in electronic communication networks and took experiments. Experimental data shows that, the computing efficiency of the system was improved by more than twice compared to traditional methods. This work not only effectively addressed the limitations of the time-domain serial acquisition method, which was time-consuming, and the limitation of the frequency-domain parallel search method, but also improved the efficiency of the weak signal capture system for electronic communication networks

Fast Fourier Transform (FFT) is an efficient algorithm to compute DFT. It can reduce the computational workload and significantly increase the computing speed of the DFT [12]. Tan found that, in the power generation system, the monitoring system and each power generation device need to obtain the real-time relevant operating status. However, the collected data usually contains a large amount of random noise or interference signals. How to quickly and accurately detect each parameter directly determines the performance of the entire system. The harmonic problem is an issue that power electronic devices have always been unable to completely solve. It has a serious impact on the safety, stability, and economic operation of the power grid. This work adopts the FFT algorithm to process the digital signal, analyze the frequency, amplitude, phase angle and other related data, and correct the relevant data to obtain relatively accurate original data.

Short-Time Fourier Transform (STFT) is used to analyse signals whose frequency would change as time over time. Hu [13] found that the former modulation type identification algorithms was difficult to adapt to the complex and changeable electromagnetic environment. These algorithms must also possess the ability to extract and analyze the subtle features of radar jamming signals to keep up with the increasing complexity of radar signals. In that case, Hu constructed mathematical models for active suppression interference and deception interference, and based on mathematical simulation, used time-frequency analysis methods to analyze the collected interference signals for feature analysis. Thus, this work verified the simulation results. Finally, it was found that the time-frequency analysis results of linear frequency modulation information could determine the interference pattern, and the interference parameters could be accurately determined. Zhou et al. [14] discovered that multi-meter electrical meter boxes had problems of incorrect detection and omission in the detection of string households. To address these issues, this work proposed an intelligent detection method for string households of multi-meter electrical meter boxes based on spectrum signal analysis. The work used voltage sensors to collect voltage signals from multi-meter electrical meter boxes, and applied short-time Fourier transform to enhance the voltage signals. By analyzing the frequency spectrum signals of multi-meter electrical meter boxes, the work extracted the characteristics of string households' signals of multi-meter electrical meter boxes. Based on the signal characteristics, the work estimated the string households score of the electrical meter boxes and identified the string households' behavior of multi-meter electrical meter boxes, achieving high-

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precision intelligent detection of string households of multi-meter electrical meter boxes. This provided strong theoretical support for the detection work, improved the detection accuracy, and solved the problems of low efficiency and high error rate in manual detection.

To sum up, different variants of Fourier Analysis could help process signals, which could improve the efficiency and accuracy of work.

3.3 Application in testing products

In the real life, many testing methods has been applied into use for many years. However, some current methods exist many limitations, for example, insufficient consideration, vulnerable to other factors and hard to handle samples. Infrared spectroscopy is also used in some detection methods, but it is not yet perfect. Fourier Analysis could help test products more comprehensive and quicker.

Fourier Series (FS) is an effective tool for analysing periodic signals, it can decompose the signal into harmonic components that contain different frequency components [15]. Wang and Li believed that the traditional safety assessment of blasting vibration did not take sufficient considerations into account. Therefore, their work proposed an amplitude-phase joint waveform simulation method based on Fourier analysis to meet the needs of blasting vibration analysis and safety assessment. This method can more accurately characterize the distribution and variation of each frequency component and provide more accurate input loads for fine analysis of dynamic response of complex structures.

Fourier Transformation Infrared Spectroscopy (FTIR) has demonstrated remarkable advantages in product testing across industries such as cosmetics [16], food [7], and materials [17, 18], owing to its rapid, non-destructive, and highly sensitive characteristics. Qian et al. [7], based on previous studies, focused on reviewing the principles of Fourier Transform in infrared spectroscopy, the data processing methods in chemometrics, and their applications in food quality safety detection. Infrared spectroscopy can reveal information about molecular chemical bonds and is highly efficient and accurate in food rapid analysis, so it is very suitable for analyzing food systems with a large number of complex molecules. By combining with chemometrics methods, qualitative and quantitative analysis of substances can also be achieved. Li et al. [16] believed that physical exfoliating cosmetics often contain plastic microbeads, which are difficult to degrade and can cause environmental pollution. These plastic microbeads would also affect human health through the food chain. Among the existing detection methods, infrared spectroscopy is also commonly used but has problems such as complex sample processing. To improve the qualitative detection efficiency of abrasive particles in cosmetics, based on the self-developed rinsing and filtration device, the cosmetics are dispersed using an appropriate dispersing solvent, the target abrasive particles are extracted, and a standard sample spectral library is established using a Fourier transform infrared spectrometer to screen the target substances of the samples. This method is simple, rapid, accurate and efficient, and is suitable for the rapid screening and identification of prohibited plastic microbeads and some abrasive components in cosmetics. Han et al. [17] believed that traditional testing methods were difficult to accurately extract the distillate fractions of different grades of base liquor and were also affected by environment temperature. Their work utilized Fourier transform mid-infrared spectroscopy combined with partial least squares method to establish a rapid detection model for 8 key flavor substances in strong-flavor base liquor and solved the interference of environmental temperature differences on detection accuracy through the global correction method with temperature compensation. The temperature compensation global correction method solved the problem of low accuracy of the detection system caused by large changes in environmental temperature. Silicone rubber will age over time under harsh conditions, resulting in a decline in

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performance and posing a threat to the safety of power systems. Li et al. [18] believe that all traditional aging assessment methods have their own limitations and that new methods are needed. Their work conducted FTIR tests to analyze the effects of different aging conditions on the main chains and side chains of high-temperature vulcanized silicone rubber and liquid silicone rubber, and inferred the changes in their hydrophobicity, mechanical and electrical properties.

To sum up, Fourier analysis, especially FTIR, can be used to testing products rapidly and efficiently.

4. Conclusion

As a core tool in processing image, Fourier Analysis could help people to analyse both periodic and aperiodic signals. By transforming signals from time domain into frequency domain, Fourier Analysis helps people to deal with many problems, making people's work more efficient and accurate. It can combine with other method to help people identify objects that seems to be similar quickly and accurately, making people's identification work easier. It is also non-destructive and environmentally friendly, causing no pollution to the atmosphere. It can improve the speed of signal processing, addressing the limitation of traditional methods. The efficiency and accuracy of signal processing would also be improved. Fourier Analysis would also assist people testing and evaluating products. It can combine with other method to make the accuracy of detection higher. But there are still some limitations and many fields that people have not perfected and studied, for example further applying the improved methods into more fields, promoting the speed of signal processing. Many methods may be not able to adapt and be used in changing environment. The accuracy of identification and detection may be affected by the environmental factors. Fourier Analysis and its derivative methods would be further studied, improved and applied into more fields.

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