Proceedings of the 4th International Conference on Software Engineering and Machine Learning
DOI: 10.54254/2755-2721/2026.32744

Backlash Calculation of Involute Small Tooth Number Difference Transmission System

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Abstract. Backlash of involute planetary drive with small tooth difference is significantly affected by machining and assembly. This paper analyzes main error sources including tooth thickness deviation, center distance error and bearing clearance, establishes individual backlash calculation models and converts them uniformly to the output end, classifies errors according to normal and Rayleigh distributions, and conducts probability simulation of the overall system backlash using the Monte Carlo method.

Keywords: involute small tooth difference drive, planetary reducer, backlash calculation, error analysis, Monte Carlo simulation

1. Introduction

Owing to large transmission ratio, compact structure and high load capacity, involute planetary drive with small tooth difference is widely used in robots and processing equipment. In engineering, affected by various working conditions such as machining accuracy and assembly dimensional chains, the accuracy and stability of transmission backlash cannot be guaranteed. Therefore, it is of great theoretical significance to establish a backlash analysis model for the transmission system. Extensive research has been conducted on errors and backlash in precision reducers: Ma Dongsheng et al. [1] analyzed multiple errors of RV reducers and proposed that combining positive equidistant and negative shift modification reduces errors; Ren Zhongyi et al. [2] modeled and simulated manufacturing and installation errors, providing a basis for accuracy improvement; Wang Huiliang et al. [3] built an RV reducer error model, clarifying the relationship between original and transmission errors for precision design; Liang Shunke et al. [4] performed finite element stress-strain analysis and proposed reasonable cycloid gear modification to improve accuracy; Qiu Tao et al. [5] established a rigid-flexible coupling model, identifying eccentric shaft torsional deformation as the main error source; Zhang Yueming et al. [6] found output disc shaft hole eccentricity has the most significant effect; Xu Hang et al. [7] developed a backlash model and systematically analyzed key influencing factors and error combination rules.

2. Overview of backlash in involute small tooth difference transmission systems

When an involute planetary gear drive with small tooth difference reverses direction, clearances exist in the meshing pairs due to component tolerances, gear tolerances and assembly dimensional

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chains. When the input shaft rotates forward, the output planet carrier lags behind; when the reducer reverses, the output angle lags relative to the input, and this lag is defined as transmission backlash.

According to the definition of backlash in gear transmission systems, it can be expressed by the following formula:

$$B = \frac{\theta_1}{i} - \theta_2 \tag{1}$$

In the formula: B denotes the backlash of the gear transmission mechanism, θ_1 denotes the input angle during reverse rotation, i denotes the transmission ratio of the gear transmission mechanism, and θ_2 denotes the rotation angle of the output planet carrier at the corresponding instant.

3. Overview of backlash in involute small tooth difference transmission system

3.1. Error sources

This paper analyzes and introduces the influencing factors of backlash in the designed involute gear transmission system with small tooth difference, and then investigates the effects of different error sources on backlash.

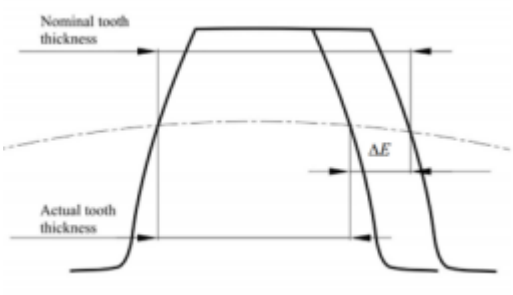

Figure 1. Schematic diagram of tooth thickness deviation

As shown in the figure1, the circumferential backlash j_{Es} caused by tooth thickness deviation is expressed as:

$$j_{Es} = \Delta E \tag{2}$$

Then the formula for backlash caused by tooth thickness deviation is as follows:

$$B_{Es} = \frac{j_{Es}}{1000 \bullet r_i} \times \frac{180}{\pi} \times 60 = \frac{6.876 \times j_{Es}}{m_i z_i} \tag{3}$$

In the formula: m_i is the module of the corresponding gear (mm), z_i is the number of teeth of the corresponding gear, j_{tEs} is the circumferential backlash caused by tooth thickness deviation, and 6.876 is the conversion coefficient for arcminutes.

3.2. Center distance error

Theoretically, the center distance of the gear pair is consistent with the theoretical value. However, during actual installation, affected by installation errors, pitch circle runout and other factors, there

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is a deviation between the actual center distance and the theoretical value, which is defined as gear pair center distance error Δa .The circumferential backlash $j_{\Delta\alpha}$ caused by this deviation is shown in Figure 2.

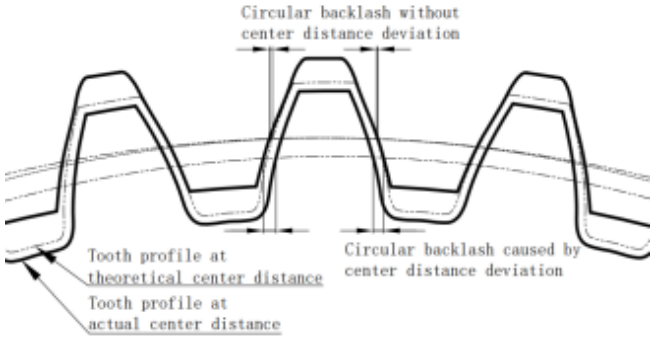

Figure 2. Circumferential backlash induced by center distance error

During meshing of a gear pair, if maximum radial center distance deviation Δa occurs at one end with increased circumferential backlash, the backlash on the opposite side decreases while the total circumferential backlash at the reference circle remains constant. The expression for backlash $j_{\Delta a}$ is:

$$\frac{j_{\Delta a}}{2} = \Delta a \bullet tan\alpha_n \tag{4}$$

The backlash caused by center distance deviation is expressed as:

$$B_{\Delta a} = \frac{6.876 \times j_{\Delta a}}{m_i z_i} \tag{5}$$

3.3. Radial clearance of bearing

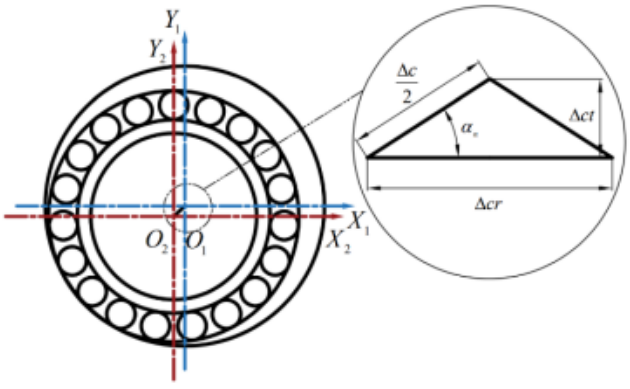

Figure 3. Schematic diagram of bearing radial clearance

Under ideal conditions, the inner and outer rings of the bearing rotate coaxially with clearance Δc , and the clearance at all positions is $\Delta c/2$. In actual operation, the bearing inner ring moves along the meshing line with the rotation of the crankshaft. As shown in Figure 3,bearing clearance $\Delta c/2$ can be decomposed into radial offset Δcr and axial offset Δct .The axial offset corresponds to

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the gear circumferential backlash caused by bearing radial clearance, while the radial offset is the center distance offset. Therefore, the expression for the circumferential backlash at the reference circle caused by bearing radial clearance is:

$$\begin{cases} \Delta ct = \Delta c \bullet cos\alpha_n \\ \Delta cr = \frac{\Delta c \bullet sin\alpha_n}{2} \\ j_{\Delta c} = j_{\Delta cr} + j_{\Delta ct} = \Delta ct + 2\,\Delta\, cr \bullet tan\alpha_n \end{cases} \tag{6}$$

The backlash caused by the gear circumferential backlash induced by bearing radial clearance can be expressed as:

$$B_{\Delta c} = \frac{6.876 \times j_{\Delta c}}{m_i z_i} \tag{7}$$

3.4. Bearing hole position deviation

During operation of an involute small tooth difference planetary reducer, the theoretical rotation center of the planet gear is O_1 .Affected by bearing hole position deviation caused by manufacturing errors, the actual rotation center is O_2 .

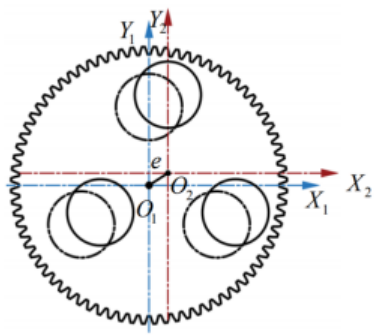

Figure 4. Schematic diagram of bearing hole position error

As shown in Figure 4, the deviation between theoretical and actual center distances produces geometric eccentricity. When the planet gear meshes with the internal gear ring, this eccentricity varies with the rotation angle. Thus, the circumferential backlash caused by bearing hole position deviation is:

$$j_e = 2K_\alpha \bullet tan\alpha_n \bullet \left(\frac{e}{2}\right) sin\theta \tag{8}$$

Then the formula for backlash caused by bearing hole position deviation is:

$$B_{\Delta e} = \frac{6.876 \times j_{\Delta e}}{m_i z_i} \tag{9}$$

3.5. Gear radial runout error

Gear radial runout error is mainly caused by geometric eccentricity error of the machine tool during gear machining. As shown in Figure 5, when geometric eccentricity exists, the deviation between the

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operating center and geometric center of the gear is e.

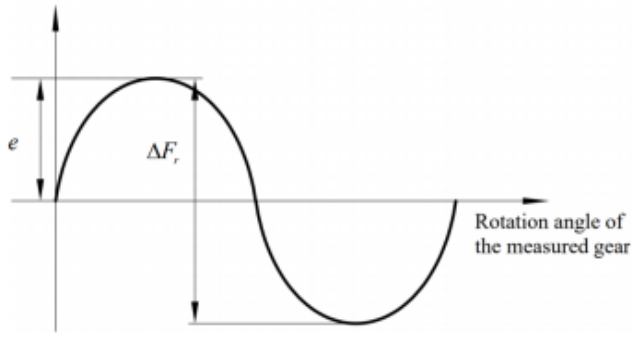

Figure 5. Radial runout tolerance of gear

As shown in Figure 5, since radial runout error 1 varies periodically with gear rotation angle, the circumferential backlash of the gear pair caused by gear radial runout error is expressed as:

$$j_{\Delta F_r} = 2K_\alpha \bullet tan\alpha_n \bullet \Delta F_r sin\theta \tag{10}$$

The backlash of the planetary gear caused by gear radial runout can be expressed as:

$$B_{\Delta F_r} = \frac{6.876 \times j_{\Delta F_r}}{m_i z_i} \tag{11}$$

The above are the main factors affecting the backlash of the reducer transmission system. Based on the analysis of the above backlash-influencing factors in this paper, the deviations of each gear and component are uniformly converted to the offset of the output planet carrier to calculate the total backlash. Since the high-speed stage components are connected to the crankshaft, their conversion to the output planet carrier must go through the low-speed stage transmission, while the low-speed stage deviations can be directly converted to the output planet carrier.

The backlash of the high-speed stage components can be expressed as:

$$B' = \frac{B_i}{i_{aH}^b} \tag{12}$$

4. Probability distribution simulation of reducer backlash

4.1. Analysis of backlash probability distribution caused by constant error terms

As shown in the analysis in Section 3, the constant error terms that do not vary with system rotation include tooth thickness deviation, gear pair center distance deviation, and bearing radial clearance, all of which follow a normal distribution and take values within the range; when the probability distribution is within the 3σ confidence interval, the distribution characteristics are as follows:

$$\begin{cases} f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] = 0.997 \\ \mu = \frac{x}{2}, \sigma = \frac{x}{6} \end{cases} \tag{13}$$

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Based on the probability distribution model, programming simulation is conducted to generate 100,000 random samples for each error term according to the typical distribution and numerical characteristics. The backlash probability distribution caused by constant error terms is calculated based on the additivity of normal distribution. Images are exported after running, and its distribution probability histogram is shown in Figure 6.

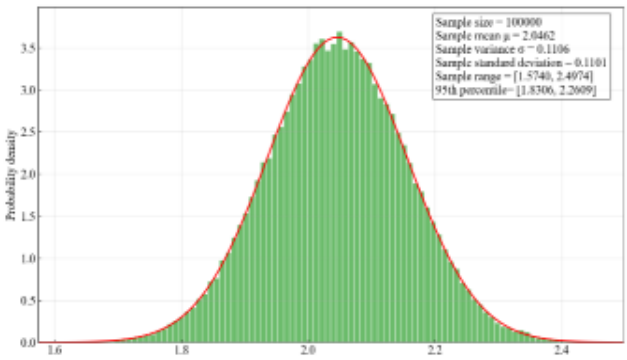

Figure 6. Probability distribution of backlash caused by constant error terms

The sample size of constant error terms is 100,000, with sample mean μ of 2.0462, variance σ of 0.1106, standard deviation of 0.1101, sample range of [1.5740, 2.4974], and 95% quantile of [1.8306, 2.2609].

4.2. Analysis of backlash probability distribution caused by variable error terms

Variable error terms vary with the rotation phase angle of the system, including bearing hole position error and radial runout error, which follow the Rayleigh distribution; when the probability distribution is within the 99.7% confidence interval, the distribution characteristics are as follows:

$$\begin{cases} f(x) = 1 - exp\left(-\frac{x^2}{2\sigma^2}\right) = 0.997 \\ \sigma = \frac{x}{\sqrt{-2ln(1-0.997)}} = \frac{x}{3.4} \end{cases} \tag{14}$$

Similarly, computer-aided simulation is adopted to calculate the backlash caused by error terms following the Rayleigh distribution, and 100,000 sample parts are randomly generated. According to the program calculation results, the backlash probability distribution caused by variable error terms is shown in Figure 7.

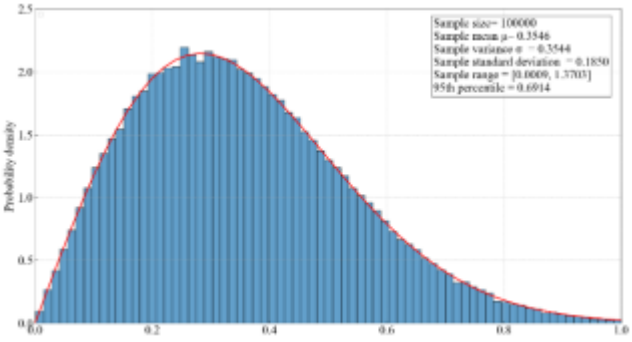

Figure 7. Probability distribution of backlash caused by variable error terms

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4.3. System backlash distribution simulation based on Monte Carlo method

System errors follow normal distribution and Rayleigh distribution, which cannot be directly superimposed to calculate the total error. Therefore, the Monte Carlo method is used to simulate the total backlash. Based on probability and statistics, this method conducts random sampling for constant error terms and variable error terms respectively, and approximately solves the total backlash through a large number of repeated samplings and statistical calculations.

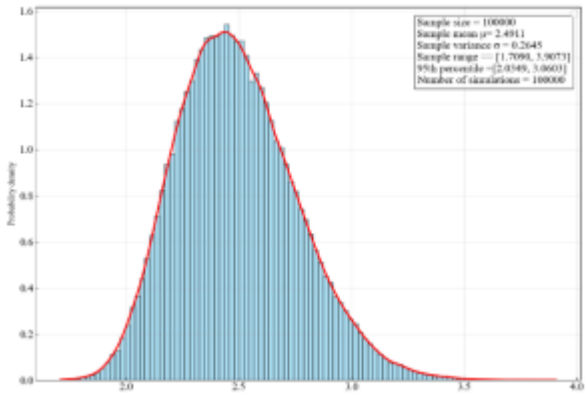

Figure 8. Probability distribution of system total backlash simulated by monte carlo method

The design requirement for maximum backlash of the small tooth difference involute planetary reducer is 3 arcmin. From Figure 3.18 and the 95% confidence interval, the maximum total system backlash is 3.0606 arcmin, with a standard deviation of 0.2648 arcmin and a mathematical expectation of 2.4927 arcmin. The 95th percentile interval is [2.0343, 3.0606] arcmin, within which 95% of products fall in 100,000 simulations, indicating high statistical accuracy. The total backlash concentrates around 2.5 arcmin, meeting machining and assembly precision requirements.

5. Conclusion

This paper systematically analyzes and calculates the backlash characteristics of an involute planetary reducer with small tooth difference, and establishes a backlash model for each component. Based on the probability distribution model, the Monte Carlo sampling method is used to calculate the total backlash, with its mathematical expectation being 2.4927 arcmin and the 95th percentile interval being [2.0343, 3.0606] arcmin. Combined with the simulation and experiment in Chapter 4, the maximum total backlash is 3 arcmin, which verifies that the calculation model is accurate, reliable and meets engineering practice, and that the machining and assembly design of the transmission system is reasonable.

References

[1] Ma Dongsheng, Tan Peng, Zhang Bin. Analysis and Experimental Research on Transmission Error Characteristics of RV Reducer [J]. Machine Tool & Hydraulics, 2021, 49(16): 145-149.

[2] Ren Zhongyi, Mao Shimin, Guo Xuedong. Accurate Modeling and Experimental Research on Geometric Backlash of RV Reducer [J]. Mechanical Science and Technology for Aerospace Engineering, 2022, 41(08): 1216-1223. DOI: 10.13433/j.cnki.1003-8728.20200453.

[3] Wang Huaqing, Wang Huiliang, Su Jianxin. Simulation Analysis of Transmission Error of RV Reducer Based on ADAMS [J]. Machine Tool & Hydraulics, 2021, 49(08): 135-140.

DOI: 10.54254/2755-2721/2026.32744

[4] Liang Shunke, Song Lei, Chen Feixin, et al. Analysis of Key Structures of RV Reducer Based on Finite Element Method [J]. Machine Tool & Hydraulics, 2019, 47(23): 165-168+115.
[5] Qiu Tao, Du Qungui. Dynamic Simulation Research on RV Reducer Based on Rigid-Flexible Coupling [J]. Journal of Mechanical Transmission, 2019, 43(12): 93-96+108. DOI: 10.16578/j.issn.1004.2539.2019.12.017.
[6] Zhang Yueming, Zhu Guilong, Ji Shuting. Error Analysis of Transmission Mechanism for RV Reducer in Industrial Robots [J]. Machinery Design & Manufacture, 2022, (05): 261-265+269. DOI: 10.19356/j.cnki.1001-3997.20211105.025.
[7] Xu Hang, Wei Chenzhou, Cui Jianghong, et al. Research on Influence of Pin Tooth Error Combination on Backlash of RV Reducer [J]. Machine Tool & Hydraulics, 2023, 51(19): 171-175.