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Coupled Electro-Elastic Dynamic Analysis of Two Fixed-End Macro Fiber Composite (MFC) Beams on the Same Side

Yuxuan Zhan

School of Sciences, Chang'an University, Xi'an, China
2023903584@chd.edu.cn

Abstract. Macro Fiber Composites (MFC) are widely used for active control and structural health monitoring of beam-slab structures due to their high flexibility and electromechanical coupling efficiency. Addressing the issue that existing research often focuses on symmetrical or cantilever configurations while neglecting the asymmetric dynamic response of single-sided MFCs on the same side of a fixed beam, this paper establishes an electro-elastic coupling dynamic model of two P1-type (d33 mode) MFCs at three equal points on the same side of the beam under the same-side MFC bonding condition, taking a two-end fixed aluminum beam as the object. Considering the neutral axis offset caused by single-sided bonding and the equivalent bending stiffness correction of the bonded segment, an electric field correction coefficient is introduced based on the electric field characteristics of the interdigitated electrodes, equating the bending electric field to a three-directional uniform electric field to describe the piezoelectric strain effect. This paper uses Hamilton's principle and the Galerkin method to derive the vibration control equations, including the mass matrix, bending stiffness matrix, and geometric stiffness matrix, and solves for the first-order natural frequencies; the theoretical results are then compared and verified with finite element simulations.

Keywords: MFC, Ipsilateral Single-sided Application, Electro-elastic Coupling, Neutral Axis Offset

1. Introduction

Macro Fiber Composite(MFC) is composed of piezoelectric ceramic fibers, polymer matrix and interdigitated electrodes (IDE) . It has high electromechanical coupling efficiency and strong flexible adhesion ability, so it is very suitable for integration with curved surface and thin-walled structures. In the late 1990s, NASA and other institutions promoted the development of MFC to overcome the shortcomings of traditional piezoelectric ceramic sheets, such as high brittleness, easy breakage and poor compatibility with curved surface structure [1,2]. With the continuous development of MFC technology, the research on its electro-elastic coupling characteristics has also become increasingly in-depth. In recent years, around the non-uniform electric field and polarization inhomogeneity induced by IDE, researchers have proposed a low-order modeling method for continuous electric field [3] and a modeling and homogenization method considering non-uniform

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polarization [4], which provides a more reliable equivalent parameter basis for structural electro-elastic coupling analysis.

MFC has a wide range of applications and has been applied and verified in aerospace scenarios such as active shape control and vibration suppression of aircraft wings and panels, intelligent control of helicopters/rotors and blades [5,6]. In the field of structural health monitoring, online damage identification based on mechanical electrical impedance (EMI) methods has continued to develop and is gradually moving towards engineering applications [7-9]. In terms of energy harvesting and self-powered sensing, MFC has also shown good application potential [10]. Domestic scholars have accumulated a relatively systematic body of research on the engineering application of MFC materials and actuators, the design and performance prediction of low-frequency sensors, etc., which provides an important reference for the integrated application of MFC in complex structures [2,7,8].

In summary, although existing research has laid the foundation for the material equivalence, structural modeling and application verification of MFC, there are still the following shortcomings : (1) Most of the work focuses on double-sided symmetrical bonding or cantilever beam structures, and lacks systematic research on the mechanical behavior of asymmetric configurations such as single-sided bonding on the same side to a fixed beam (or fixed-fixed beam). (2) Existing research mostly discusses the influence of MFC size parameters or bonding position in isolation, lacks a systematic revelation of the multi-parameter coupling mechanism, and is difficult to support engineering optimization layout. On the other hand, in order to obtain stronger modal coupling and higher control efficiency, the layout optimization of the patch/actuator has gradually attracted attention. Existing research has given the optimal layout strategy of a thin piezoelectric patch from the perspective of modal coupling [11], which provides an optimization idea that can be referenced for engineering design.

Based on the above analysis, this paper focuses on a fixed beam structure with MFC applied to one side only, and studies the asymmetric piezoelectric patch actuation mechanism in classical beam actuation and laminated beam models, incorporating existing discussions. This paper establishes an equivalent beam-pattern model considering patch asymmetry and electro-elastic coupling, aiming to construct a dynamic model of a fixed beam structure with MFC applied to one side only. The accuracy of this model is verified through comparison of theoretical and simulation results. The aim is to provide a theoretical foundation for future applications of MFC in vibration control.

2. Literature review

Regarding the acquisition of equivalent electro-elastic coupling parameters of MFC, existing studies have proposed various equivalence and parameter identification paths, such as hybrid rules and asymptotic expansion, which provide usable equivalent material constants for structural finite element modeling. Deraemaeker et al. proposed hybrid rules to predict the piezoelectric performance of MFC, providing ideas for the acquisition of equivalent material constants [12]. Biscani et al. derived the equivalent electro-elastic parameters of MFC based on the asymptotic expansion method, realizing the equivalence from micro to macro [13]. Prasath and Arockiarajan further gave an equivalent motor response model combining analytical and numerical methods for determining the input parameters of a structural finite element [14]. In terms of structural modeling and simulation, Zhang et al. established a linear and geometrically nonlinear finite element model considering fiber orientation under the Reissner–Mindlin theoretical framework, which can simulate the dynamic response of MFC laminated smart structures [15]. Domestically, Zhang Bo et al. started from the "material-device-structure" chain and revealed the dynamic behavior of the MFC cantilever

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structure and its sensitivity to boundary conditions through experiments and theoretical analysis, providing a basis for performance evaluation in engineering applications [16]. With the deepening of research, the most basic modeling has been perfected, and the next step is to consider the influence of factors such as the contact layer.

In terms of structural health monitoring and interface issues, the adhesive layer interface between the MFC piezoelectric sheet and its substrate structure will significantly affect the strain transfer efficiency and monitoring and control effects. In terms of impedance modeling, Bhalla and Soh established a mechanical impedance model for bonded piezoelectric transducers and clarified the important influence of adhesive layer parameters on coupling response [17]. Moharana and Bhalla further improved the modeling method of bonded transducers from the perspective of continuum modeling and improved the model consistency [18]. In terms of shear hysteresis effect description, Kapuria et al. proposed a dynamic shear hysteresis model suitable for bonded piezoelectric transducers, revealing the mechanism of adhesive layer shear hysteresis/inertia on transfer efficiency, and providing a theoretical basis for boundary and interface correction modeling [19]. At the same time, the viscoelastic characteristics of the adhesive layer material will change the interface shear transfer and actuation efficiency, thereby affecting the static/dynamic response of the structure. Related numerical studies have given the sensitivity law of adhesive layer viscoelastic parameters to actuation effect [20]. Recent studies have further focused on the influence of variable operating conditions and long-term service conditions on the response stability of piezoelectric transducers. Tenreiro et al. systematically reviewed the electromechanical impedance spectroscopy (EMI) method for adhesive structures, pointing out that factors such as temperature, humidity, load, and adhesive aging can cause admittance spectrum drift, and temperature compensation and feature robust extraction strategies need to be combined to improve the reliability of long-term monitoring [21]. Brunner summarized the main influence mechanisms of variable operating environment on piezoelectric transducers from the perspective of long-term structural health monitoring, and gave the corresponding suppression and correction ideas [22].

With the gradual improvement of MFC modeling and the deepening understanding of MFC hysteresis and nonlinear effects, researchers have gradually introduced more refined hysteresis modeling and adaptive control algorithms into the vibration suppression of flexible structures. For example, Gao et al. proposed a discrete-time rate-related hysteresis modeling and adaptive filtering control framework, and verified the convergence and vibration suppression effect of the control algorithm through a flexible beam test platform [23]. In more complex fluid-structure interaction structures, Zhang et al. established a semi-analytical dynamic model considering fluid-structure coupling and MFC piezoelectric effect for infusion pipelines, and carried out theoretical and experimental research in combination with control strategies such as LQR and PID, verifying the feasibility of MFC in pipeline vibration reduction [24]. In addition, in high-performance structures such as aero-engine blades, Gao et al. further carried out experimental research on piezoelectric element layout optimization and vibration suppression, providing a reference for the integration and application of MFC piezoelectric elements on complex curved surface structures [25].

3. Physical model and neutral axis correction of MFC beam structure

Studies an aluminum beam with an axially fitted P1-type (d33 mode) MFC, as shown in Figure 1. Now, taking the example of the geometric center of the MFC located at the trisection point on the same side of the beam, a series of equations is derived, and the same logic applies to other locations.

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Figure 1. Schematic diagram of the beam structure with MFC applied

3.1. Model geometry settings

The beam has a length of $L=380\,mm$, a width of $B=20\,mm$, a thickness of $C=2mm$, a density of $\rho_b=2684.1\,kg/m^3$, and an elastic modulus of $E_b=68.5\,GPa$. (Note the subscripts; the same applies below). Establish a coordinate system at the centroid of the left end section of the beam, with x, y, and z axes for the thickness, width, and length directions, respectively. Two MFC sheets are applied to one side of the beam, at positions and $z\in[z_1,z_2]$ $z\in[z_3,z_4]$. Their geometric centers are located at L/3 and 2L/3, respectively.

The relevant parameters of MFC are length $a=85\,mm$, width $b=14\,mm$, thickness $c=0.3\,mm$, density $\rho_p=5440\,kg/m^3$, piezoelectric constant $d_{33}=460\times10^{-12}\,m/V$ (high field value), and electrode spacing $t_p=0.5\,mm$. Tensile modulus $E_p=30.34GPa$

3.2. Neutral axis offset and equivalent bending stiffness

Since both MFC sheets are applied to only one side of the beam, the overall neutral axis inevitably shifts towards the applied side (top). This is the biggest difference from symmetrical applications or models that ignore thickness. Let the offset of the neutral axis relative to the geometric center axis of the beam be denoted as e. According to composite material mechanics theory, within the applied section, the position of the neutral axis is determined by the first moment of the cross-section:

$$e=\frac{E_pA_py_p}{E_bA_b+E_pA_p}=\frac{E_pbc(C+c)/2}{E_bBC+E_pbc} \tag{1}$$

Similarly, due to the neutral axis offset, the equivalent bending stiffness of the MFC-coated section $(EI)_{eq}$ will also change and needs to be corrected:

$$(EI)_{eq}=E_b[I_b+A_be^2]+E_p[I_p+A_p(\frac{C+c}{2}-e)^2] \tag{2}$$

Wherein the moment of inertia:

$$I_b=\frac{1}{12}BC^3,A_b=BC \qquad I_p=\frac{1}{12}bc^3,A_p=bc \tag{3}$$

For ease of subsequent calculations, the stiffness distribution function is defined $D(z)$ as follows:

$$D(z)=E_bI_b+[(EI)_{eq}-E_bI_b]\cdot[H(z-z_1)-H(z-z_2)+H(z-z_3)-H(z-z_4)] \tag{4}$$

Here, $H(z)$ represents the transition function, which is used only at the application section $(EI)_{eq}$ and in other areas E_bI_b.

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3.3. Electric coupling dynamics modeling

Due to the interdigitated electrode structure of MFC, there is a bending electric field between adjacent electrodes. Using the electric field correction coefficient η obtained in reference [17], the electric field is equivalent to a uniform electric field with a specific direction. The strain εp generated by MFC under the action of the electric field is expressed as:

$$\varepsilon_p = d_{33}\frac{U}{t_p} \tag{5}$$

When both patches are applied on the same side, the axial resultant force of the applied segment $N(z)$ is:

$$N(z) = \begin{cases} N_{piezo} & z \in [z_1, z_2] \cup [z_3, z_4] \\ N_{beam} & \text{else} \end{cases} \tag{6}$$

Considering that boundary conditions restrict the overall axial displacement, the equivalent axial force generated by the piezoelectricity causing the MFC to deform in coordination with the beam can be approximated as:

$$P(z) = F_{mfc} \cdot [H(z-z_1) - H(z-z_2) + H(z-z_3) - H(z-z_4)] \tag{7}$$

Wherein F_{mfc} is the effective region force generated by the constraint of MFC, approximately:

$$F_{mfc} \approx E_p b c d_{33}\frac{U}{t_p} \tag{8}$$

Considering the linear density in the application section:

$$\rho A(z) = \rho_b BC + \rho_p bc \cdot [H(z-z_1) - H(z-z_2) + H(z-z_3) - H(z-z_4)] \tag{9}$$

Total kinetic energy:

$$T = \frac{1}{2}\int_0^L \rho A(z)\left(\frac{\partial w}{\partial t}\right)^2 dz \tag{10}$$

Bending potential energy:

$$U_b = \frac{1}{2}\int_0^L D(z)\left(\frac{\partial^2 w}{\partial z^2}\right)^2 dz \tag{11}$$

Geometric potential energy:

$$U_g = \frac{1}{2}\int_0^L P(z)\left(\frac{\partial w}{\partial z}\right)^2 dz \tag{12}$$

Using the Galerkin method, the lateral displacement is expanded into a modal superposition $w(z,t) = \sum \phi_i(z) q_i(t)$. Substituting this into the energy equation and applying variational analysis to the generalized coordinates, this paper obtains the dynamic equation:

$$M\ddot{q} + (K_b + K_g)q = 0 \tag{13}$$

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Where the mass matrix M , stiffness matrix Kb , and stiffness matrix Kg are respectively:

$$M_{ij}=\int_0^L \rho_b BC\phi_i(z)\phi_j(z)dz+\int_{MFC}\rho_p bc\phi_i(z)\phi_j(z)dz \tag{14}$$

$$K_{bij}=\int_0^L E_b I_b\phi_i''(z)\phi_j''(z)dz+\int_{MFC}\left[(EI)_{eq}-E_b I_b\right]\phi_i''(z)\phi_j''(z)dz \tag{15}$$

$$K_{gij}=\int_0^L P(z)\phi_i'(z)\phi_j'(z)dz \tag{16}$$

The first natural frequency can be obtained using the above formula. This paper changes the voltage and compares the theoretical and simulated values to verify the correctness of the theoretical calculations and the realism of the simulation model.

Table 1. Comparison of first-order natural frequencies of MFC-fixed beam structures under different voltages

U/v	Theoretical Result/Hz	FEM/Hz	e FEM
0	68. 82	68.53	-0.42 %
400	66.16	65. 67	-0.74 %
800	63. 38	62. 86	-0.82 %
1200	60. 48	60. 33	-0.24 %

The data in Table 1 shows that the frequencies obtained by the two methods are quite consistent, suggesting that the results of the theoretical derivation are reliable.

4. Conclusion

The differential equation of motion for a fixed beam structure using Hamilton's principle, considering the neutral axis shift caused by MFC (Mechanical Fusion Mixing) application on the same side, and solves for the first-order natural frequency of the fixed beam under this MFC application mode using the Galerkin method. The theoretical calculation values of the fixed beam frequency under a specific voltage are compared with the corresponding simulation results. The results are quite close, indicating that both the theoretical method and the simulation model are correct. This provides theoretical assistance for future research on structures with MFC applied to beams.

However, the electro-elastic coupling dynamic model of same-side dual MFC bonding established in this paper within the framework of Euler–Bernoulli beam theory and energy method still has limitations in terms of the analysis object and working conditions. First, this paper only considers the bonding case at three equal division points, without considering other asymmetric general cases. Second, this paper uses a correction coefficient to treat the electric field of the interdigitated electrodes as equivalent, and the influence of electric field non-uniformity and material/electrode geometric details on the coupling effect is moderately simplified. In future research, based on the existing segmented equivalent modeling approach, a unified electro-elastic coupling model applicable to arbitrary bonding positions, lengths, and numbers, asymmetric bonding, and different boundary conditions can be constructed to derive universal laws. At the same time, adhesive layer shear hysteresis and viscoelastic effects and interface damage evolution can be introduced to establish a "beam-adhesive layer-MFC" transmission mechanism model, and key equivalent parameters can be identified and calibrated through experimental modal analysis and

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electro-response testing. After considering the specific transmission mechanism model of MFC bonding, the influence of the flexibility of the fixed clamp and manufacturing errors on the results can also be considered, thus giving the applicable range and error range of the model. Ultimately, this research method can be extended to complex structures such as plates and shells, as well as application scenarios such as vibration suppression and structural health monitoring, providing more comprehensive theoretical support for the engineering application of piezoelectric smart structures.

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