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Minkowski Diagrammatics: A Computable Geometry for Lorentz-Equivariant Modeling

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Abstract. Special relativity is often introduced through algebraic formalism, yet many of its core ideas—time dilation, length contraction, and the relativity of simultaneity—are inherently geometric and can feel counterintuitive to beginners. This paper develops a visualization-first pathway that pairs spacetime diagrams with concise animations to make the kinematics of inertial frames perceptible and internally consistent. After motivating the role of diagrams in scientific reasoning, we construct frame-aware figures for three classic thought experiments: the light clock, the ladder–barn paradox, and the train-and-lightning scenario. Each case is presented with coordinated axes [x, ct] and [x′, ct′], light-cone guides, and simultaneity slices to reveal how invariant light speed and Lorentz structure constrain what observers in relative motion can agree upon. The light-clock diagram shows why moving clocks tick slower by tracing longer null-bounded paths between mirror strikes; the ladder–barn construction demonstrates length contraction as a frame-specific "slice" through a worldsheet; and the train–lightning example renders frame-dependent simultaneity without contradiction. We discuss affordances [conceptual scaffolding, error diagnosis, and transfer to algebra] and limitations [idealization, static figures for accelerated motion] and outline a design space for interactive tools that link diagrams, algebra, and simulation. The result is a compact, teachable sequence that helps novices "see" relativistic effects while preserving mathematical rigor.

Keywords: Special Relativity, Minkowski Geometry, Lorentz-Equivariant Machine Learning, Spacetime Diagrams

1. Introduction

Special relativity is fundamentally geometric: inertial kinematics is governed by the Lorentz structure of Minkowski space, where invariants [intervals, causal order, proper time] constrain any admissible account of events. Yet most computational pipelines impose these constraints post hoc, treating diagrams as illustrations rather than first-class mathematical objects. This paper advances a visualization-first program in which spacetime constructions—worldlines/worldsheets, simultaneity foliations, and light-cone generators—are encoded as computable state, enabling direct translation between geometry and algebra and furnishing symmetry-aware targets for numerical methods and machine-learning models.

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In special relativity, the structure of spacetime defies common sense. Time and space are now intertwined, and measurements depend on motion. Visual representations become essential in making geometric relationships visible rather than relying solely on equations [1]. For students, this is especially significant. Diagrams in special relativity not only illustrate; they provide many given numbers and explain situations provided in words in graphs, which serves as a means through which complex theoretical structures become approachable [2].

Moreover, diagrams in physics are more than a tool. They many times are integral to the reasoning process. Visualizing a problem is a big portion in scientific thinking itself [3]. Experts in this field often create diagrams to face a problem, organize conceptual understanding, and guide a solution. A similar situation can be observed in the context of teaching relativity. Diagrams allow learners to see how time dilation, length contraction, and simultaneity in difference frames can be explained by geometric relations. This transforms abstract formulas to a visual that can be grasped intuitively [4].

This paper explores how diagrams can make special relativity easier to understand, focusing on three classic thought experiments such as the ladder-and-barn paradox, the light clock, and the train and lightning experiment. These scenarios involve events that can be seen differently but all make sense in different frames, since distance and time are measured differently. The aim is to fill the gap between abstract math and intuitive understanding, making concepts such as time dilations and length contraction more approachable for highschoolers.

Spacetime diagrams and short animations show both how these scenarios are seen visually and mathematically. In the spacetime diagrams, the two objects' paths and the two frames are shown at the same time, and the tilt of the axis and the line of movement for objects will be calculated with formulas. These tools will help accurately represent the event.

To make the approach concrete and testable, we develop three canonical benchmarks—the light clock, ladder–barn, and train-and-lightning setups—as model problems spanning analytic derivation, numerical specification, and symmetry-preserving evaluation. Each benchmark provides parameterized worldsheets, invariant checks, and frame transforms suitable for optimization or learning tasks that must remain Lorentz-equivariant. This reframing positions visualization as a geometric calculus for special relativity, aligning with downstream goals in computational physics and machine learning rather than pedagogy.

2. Role of visualization

Visualization has always been playing a significant role in physics. Before the use of computer models and simulations, scientists used geometric shapes and diagrams to test out ideas and find solutions to problems. In classical mechanics, an example can be the free-body diagram. It helps identify forces and predict motions. Another example is ray diagrams in optics, which makes it possible to see how light behaves. In many cases, drawing is a part of the reasoning process itself: these diagrams do not just illustrate existing ideas; ideas also emerge through these means [5].

Physics is not only a mental exercise in manipulating numbers. It is a science with drawing enabling scientists to process things visually. Judith Fan states that making graphical representations can improve scientific reasoning by helping to organize information, gain insight, and communicate complex ideas [6]. On the educational side, physics students often draw a diagram even when they are not asked to do so, using them to orient themselves to problems [3]. More studies on science education notes that visuals enable researchers to comprehend complex and non-observable phenomena, which can be common in special relativity [7]. In all the cases previously mentioned, graphing is a way of thinking.

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As mentioned, there are many examples that require visuals to organize or generate ideas. Special relativity relies even more on visuals: many effects cannot be directly observed the way that can be observed in classical physics, since different observers disagree on what happened, and when and where an event is happening. Observers do not agree on simultaneity or even the spatial length of the same object, thus these phenomena must be identified through representations that include different frames. This is where a spacetime diagram is helpful.

A spacetime diagram has special axes, with the vertical axis showing the time [ct, with c representing the speed of light and t is the time] and the horizontal axis showing the distance [x]. In this setting the path of a light ray will always appear at 45deg angles. Two lines showing light rays divide the graph into four sections. The 2 sections colored blue are timelike regions, and the remaining 2 are spacelike regions [faster than light objects].

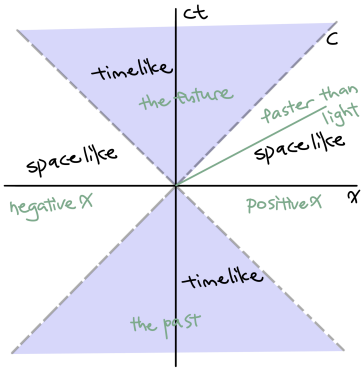


Figure 1. The basic setup of a spacetime diagram

Figure 2 shows a spacetime diagram that includes 2 observers. The axes are x and ct for a stationary observer. The manipulatable number “a” is the velocity of a moving observer [with axes ct’ and x’] in some fraction of c. When the velocity of the moving observer increases, the axes will move closer to the 45deg line that shows positive light speed.

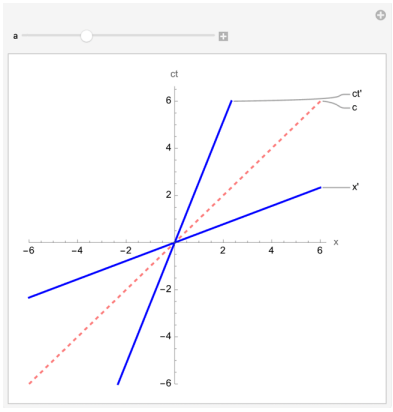


Figure 2. The behavior of the second set of axes based on the relativity velocity

In special relativity, space and time are not separate things. Like points in space can be connected with lines, events in different times and locations can be connected with worldlines. The angles, lengths, and intersections of those lines can tell many physical facts. As Taylor and Wheeler noted, once a diagram is correctly drawn, “the geometry holds the answer” [8]. The lines on these diagrams

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are called worldlines; the lines can show the location of an object, where it's traveling and how fast it's traveling.

In Figure 3, a few worldlines are shown. Because of the special settings of spacetime diagrams, the steeper a line is, the slower the object is moving. For example, Object A is slower than Object B. Object C is a stationary object. In the diagram it's a line parallel to the ct axis. In the same graph, Object D creates a parallelogram. This is an object that covers some distance, and the shape created is called a worldsheet.

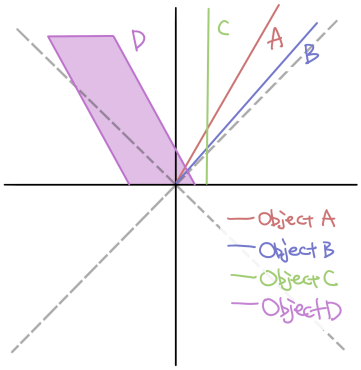


Figure 3. The worldlines and worldsheets of four objects

When worldlines are placed in a diagram, especially when the two observers' frames are overlaid using tilted axes, one can immediately see whether two observers agree on the timing of an event.

There are several ideas that can be hard to understand just seeing the calculations, but it will be much clearer when explained with a picture. An important concept is time dilations. Mathematically, it's found using a Lorentz factor, but an example and a picture show why it happens. A light clock, which will also be explained in more detail later, is a good example. A moving clock with light bouncing between 2 mirrors will make the light travel more distance than the stationary clock within one tick. This phenomenon can be explained if the time between ticks of the moving clock is longer than that of the stationary clock.

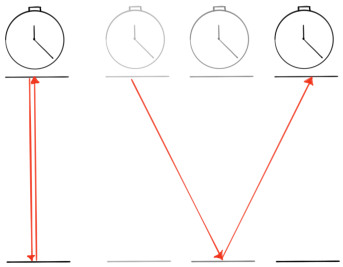


Figure 4. The light clock experiment

Similarly, length contraction becomes more visually intuitive when considering how the length of an object is measured as a "slice" in the spacetime diagram. In a frame where the observer is moving with the object, the object is one length; on the other hand, in a stationary observer's frame, a "slice" of the moving object at one specific time appears shorter than the object's original length. In a

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spacetime diagram, the path of the moving object is shown as a deformed parallelogram. Therefore, a slice is somewhat shorter than the original length as in figure.

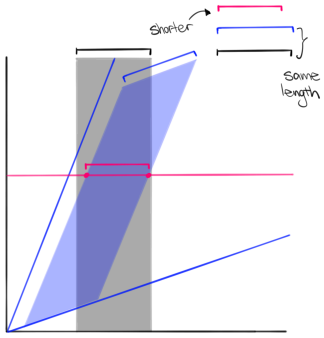


Figure 5. Effect of relative velocity on worldsheets

For all these reasons visuals are helpful in special relativity, the following sections present 3 classic thought experiments, how they are visualized, and examine how physics becomes clear when reconstructed visually. Having formalized visualization as a geometric calculus on Minkowski space, the next section instantiates this program on three canonical benchmarks—the light clock, ladder–barn, and train–lightning setups—treating them as model problems for [i] analytic derivation via Lorentz geometry, [ii] numerical validation with invariant quantities [proper time, interval, causal order], and [iii] ML evaluation of symmetry-preserving representations that must respect light-cone structure and simultaneity slices under coordinate changes.

3. Discussion

3.1. The light clock experiment

The main set up contains a clock, two mirrors that allow the light to bounce between them. There is a stationary clock and a moving one. In the light clock experiment, the moving clock's light pulse travels a longer diagonal path than the stationary one, yet the speed of light remains the same in both frames. Therefore, it can only be explained if the light has more time to travel the longer diagonal distance. But importantly, this argument depends on the fact that nothing can exceed the speed of light. If a signal or object were to travel faster than light, the order of cause and effect in some frames of reference will be reversed, which would make science itself impossible [8].

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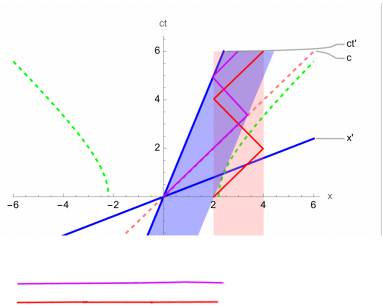


Figure 6. Spacetime diagram for light clock experiment. The purple line is a little longer than the red line

Figure 6 is a spacetime diagram. The pink rectangle is in the stationary frame [x and ct], it sets the 2 units apart boundaries where the red line bounces between. The blue axes [x' and ct'] show the frame of the moving clock. The base length of the blue parallelogram is the space between the moving clock boundaries at rest [2 units]. The purple line is the moving clock that bounces between boundaries separated by the same distance, but it covers more distance than the red line. This diagram explains why a moving clock experiences time dilation: each "tick" of the moving clock corresponds to a longer interval than a tick of the stationary clock.

3.2. The ladder-barn paradox

The setup of the ladder-barn paradox is more complex, including a stationary barn with front and back doors and a ladder carried by a moving observer running at high speed through the barn. When measured at rest, the ladder is longer than the barn. The paradox is that the ladder cannot fit in the barn, but as it's moving at a high enough speed, it seems to fit in the barn in one observer's frame but not the other. Studies show that students often misunderstand simultaneity and struggle to reconcile perspectives from different frames, which makes visual tools essential [9]. In the graph, the barn is set to be 2 units wide, and the ladder is set to be 2.2 units long. The pink rectangle indicates the stationary object [the barn] and the blue parallelogram shows the moving object [the ladder]. The black line shows a simultaneity slice; it cuts out the front and back of the ladder "simultaneously" according to the barn's frame. Due to the tilt of the axes, the base of the shape [which is the original length of the ladder] seems shorter on a slice that is only simultaneous in the frame of the barn. In the frame of the ladder or the one carrying the ladder, the two ends of the ladder are not measured at the same time. In fact, one end of the ladder is first recorded, causing the ladder to seem shorter.

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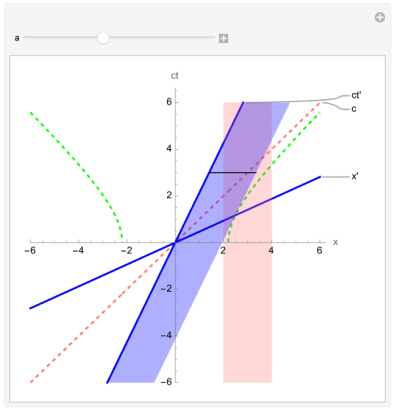


Figure 7. Spacetime diagram of ladder-pole paradox

3.3. The train-and-lightning experiment

The train-and-lightning experiment shows the relativity of simultaneity. In the set up of this experiment, there is a train moving at high speed past a platform. Bolts of lightning strike the two ends of the train. To an observer on a platform, who is standing at the midpoint of where the two bolts of lightning hit, the two bolts lightning strike the train at the same time since light travels at the same speed in all directions. However, to the observer standing in the middle and on top of the moving train, the situation is different. This observer is moving towards the front lightning and moving away from the back lightning. This causes the observer on the train to conclude that the strike at the front end happened slightly earlier than the strike at the back end. This does not mean anyone is wrong, it just points out that simultaneity is frame dependent.

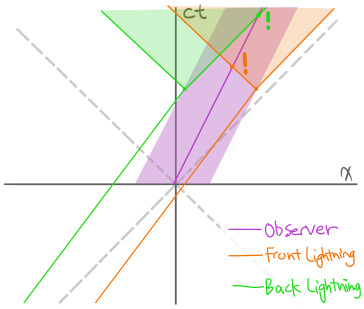


Figure 8. Spacetime diagram of train-and-lightning experiment

Figure 8 is a spacetime diagram of the train-and-lightning experiment. The light purple parallelogram shows how the train is moving; at the middle is the observer on the train. The green line is the bolt of lightning that strikes the back of the train, and the orange line is the one that strikes the front. Light is expanding in every direction, so light expands to left and right at an angle of 45 degrees. The observer will see the lightning when the line intersects with the observer line, as marked with orange and green exclamation marks.

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4. Conclusion

This paper presented a visualization-first formalism in which spacetime diagrams are not illustrations but computable objects: worldlines/worldsheets, simultaneity foliations, and light-cone constraints define a Lorentz-consistent state space onto which classic thought experiments are projected. In this setting, time dilation, length contraction, and frame-dependent simultaneity emerge as geometric invariants and slicing operations, giving a compact route from diagram to equation and back. The light-clock, ladder–barn, and train–lightning cases serve as stress tests for this bidirectional pipeline, demonstrating how "the geometry holds the answer" while remaining numerically tractable for downstream algorithms.

Three research threads follow naturally: First, Mathematical structure: extend from 2D [x,ct] sketches to higher-dimensional world-tubes and accelerated motion using differential geometry on curved embeddings; characterize invariants with category-theoretic morphisms between frames. Secondly, algorithms that build differentiable diagram generators and Lorentz-equivariant layers [SO^+ [1,3] representations, hyperbolic/Einstein-velocity additions] to enforce symmetry in learning systems; couple with physics-informed losses on intervals, causal cones, and proper time. By the very end, applications that use these benchmarks for inverse problems [event reconstruction], uncertainty quantification under frame transforms, and data synthesis for ML models that must reason about causality and simultaneity in relativistic regimes.

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